XVIII SLALM
16-20 December, 2019
Universidad de Concepción
Concepción, CHILE

http://slalmxviii.udec.cl

MODEL THEORY

Invited talks

Contributed talks

INVITED TALKS

▶ ITAÏ BEN YAACOV, *Reconstruction of non countably categorical theories.*
Institut Camille Jordan, Universit Claude Bernard – Lyon 1.
URL Address: `http://math.univ-lyon1.fr/~begnac/`

It is a classical result of model theory (known by some as the Ahlbrandt-Ziegler principle, even though it predates their paper) that automorphism groups classify complete countably categorical theories up to bi-interpretation. We propose a generalisation to the classification of arbitrary theories in a countable language (in classical logic) up to bi-interpretation by Polish groupoids based on the Cantor space. While there exist superficially similar result in categorical logic, our constructions are quite different, and are much closer to the countably categorical case.

Given enough time, I shall also discuss possible generalisations to continuous logic and potential applications.

▶ ALEXANDER BERENSTEIN, *Lovely pairs and H-expansions of geometric theories.*
Department of Mathematics, Universidad de los Andes, Bogotá, Colombia.
E-mail: aberenst@gmail.com.

A complete theory T is called geometric if the algebraic closure has the exchange property in all models of T and the theory eliminates the quantifier exists infinity. In such theories there is a rudimentary notion of independence given by algebraic independence. Examples of geometric theories include SU-rank one theories and dense o-minimal theories. An expansion of a model M of T by a unary predicate H is called dense-codense if for every finite dimensional subset A of M and every non algebraic type $p(x)$ over A, there is a realization of $p(x)$ in $H(M)$ and another one which is not algebraic over $AH(M)$. A dense-codense expansion is called an H-structure if in addition $H(M)$ is algebraically independent and it is a lovely pair if $H(M)$ is an elementary substructure of M. In this talk we will talk about the basic properties of H-structures and lovely pairs and how the new structure can be understood as a tame expansion of the original structure. This talk includes joint work with E. Vassiliev and D. Garcia and T. Zou.

▸ MIGUEL CAMPERCHOLI, *Algebraic functions.*
Facultad de Astronomía, Matemática, Física y Computación, Universidad Nacional de Córdoba, Argentina.
E-mail: mcampercholi@gmail.com.

Let **A** be an algebraic structure, and consider the system of equations

$$\alpha(\bar{x},\bar{z}) := \begin{cases} t_1(x_1,\ldots,x_n,z_1,\ldots,z_m) = s_1(x_1,\ldots,x_n,z_1,\ldots,z_m) \\ \qquad\qquad\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots \\ t_k(x_1,\ldots,x_n,z_1,\ldots,z_m) = s_k(x_1,\ldots,x_n,z_1,\ldots,z_m) \end{cases}$$

where t_j and s_j are a terms for $j \in \{1,,\ldots,k\}$. Suppose that for all $\bar{a} \in A^n$ there is exactly one $\bar{a} \in A^m$ such that $\alpha(\bar{a},\bar{b})$ holds. Then the system $\alpha(\bar{x},\bar{z})$ defines m functions $f_1,\ldots,f_m : A^n \to A$ by

$$(f_1(\bar{a}),\ldots,f_m(\bar{a})) := \text{unique } \bar{b} \text{ such that } \alpha(\bar{a},\bar{b}).$$

A function is called *algebraic* on **A** if it is one of the functions defined by a system of equations in the manner just described. For example, the complement function is algebraic on the two-element bounded lattice, as witnessed by the system

$$\alpha(x,z) := \begin{cases} x \wedge z = 0 \\ x \vee z = 1. \end{cases}$$

Given an algebraic structure **A**, it is easy to see that every term-function of **A** is algebraic on **A**, and that algebraic functions on **A** are closed under composition; that is, they a form a clone on A. Algebraic functions can be seen as a natural generalization of term-functions, and share some of their basic properties (e.g., they are preserved by endomorphisms and direct products).

Algebraic functions have been characterized for algebras in several well-known classes such as: Boolean Algebras, Distributive Lattices, Vector Spaces and Abelian Groups, among others. In our talk we will review these characterizations and discuss the main tools used to obtain them. We will also show how algebraic functions can be used in the study of epimorphisms and to describe intervals in the lattice of clones over a finite set.

▶ PETR CINTULA, *Towards a general theory of lattice-valued models.*
Institute of Computer Science of the Czech Academy of Sciences
Pod Vodárenskou věží 271/2, Prague 8, 182 07, Czech Republic.
E-mail: `cintula@cs.cas.cz`.

Many generalizations of the basic setting of model theory have been proposed over the years. One of the more radical departures from the classical picture was the many-valued generalization of the interpretation of predicate symbols, where relations are conceived as functions that assign, to each tuple of elements of the domain, values from a set that contains more than just the two classical values *true* and *false*.

There are at least three different origins of research on these structures: Boolean-valued models, continuous model theory, and semantics of predicate many-valued logics. There are many papers studying basic and advanced model-theoretic properties of these structures, mostly focused on particular approaches and sets of problems with different levels of generality and mathematical sophistication. However, the common trait of these approaches is the fact that the values form a **lattice.**

In my talk I propose a particular framework which could serve as a background for a general theory of lattice-valued structures and illustrate its utility by (1) a detailed analysis of possible forms of Skolem and Herbrand theorems and their interplay with compactness/finitarity and witnessed model property (a crucial notion in our setting which trivializes in classical model theory) (2) proving an analog of the omitting types theorem, and (3) establishing essential undecidability of very weak arithmetical theory.

[1] P. Cintula, D. Diaconecsu, G. Metcalfe, *Skolemization and Herbrand Theorems for Lattice-Valued Logics*, ***Theoretical Computer Science***, vol. 768 (2019), pp. 54–75.

[2] P. Cintula, D. Diaconecsu, *Omitting Types Theorem for Fuzzy Logics*, ***IEEE Transactions on Fuzzy Systems***, vol. 27 (2019), pp. 273–277.

[3] G. Baddia, P. Cintula, A. Tedder, *How Much Propositional Logic Suffices for Rosser's Essential Undecidability Theorem?*, ***Draft***.

▶ TOMÁS IBARLUCÍA, *Kazhdan's property (T) for automorphism groups of $\aleph_0$-categorical metric structures.*

Institut de Mathématiques de Jussieu–PRG, Université de Paris, case 7012, 75205 Paris CEDEX 13, France.
URL Address: https://webusers.imj-prg.fr/~tomas.ibarlucia.

Kazhdan's (T) is an important property about the unitary representations of certain topological groups, very much studied in the case of discrete and locally compact groups. In the last twenty years, the interest on Property (T) has extended to the domain of large Polish groups. Most notably, Bekka showed that the unitary group $U(\ell^2)$ has Property (T), and Evans and Tsankov proved that all oligomorphic groups—such as $\mathrm{Sym}(\mathbb{N})$, $\mathrm{Aut}(\mathbb{Q},<)$ or $\mathrm{Homeo}(2^\omega)$—have (T) as well.

I will discuss a generalization of these results showing that all Roelcke precompact Polish groups (equivalently, the automorphism groups of $\aleph_0$-categorical metric structures) enjoy Property (T), and in a particularly strong form. The proof, which I intend to sketch, is entirely model-theoretic. Moreover, unlike the proofs of the former particular cases, it does not rely on previous results of classification of unitary representations.

▸ HUGO RAFAEL DE OLIVEIRA RIBEIRO, AND HUGO LUIZ MARIANO, *The Witt ring and the Von Neumann hull of a real semigroup.*
Universidade de São Paulo, Brazil.
E-mail: hugomar@ime.usp.br.

The algebraic theory of quadratic forms with coefficients in a general field (of characteristic $\neq 2$) was born in 1937 by the hands of E. Witt: he has defined a ring formed by classes of isometry of non-singular and anisotropic quadratic forms; this ring has strong connections with the space of possible orderings on the base field. The notion of special group (SG) was introduced by M. Dickmann and F. Miraglia in the middle of 1990 as a first order-axiomatization of the algebraic theory of quadratic forms given by the invertible coefficients of a ring with "many units": since this axiomatization is based on the primitive concept of binary isometry, a Witt ring construction is naturally available. Almost a decade latter, M. Dickmann and A. Petrovich introduced a generalization of the concept of reduced special group (RSG): it is another first-order notion called real semigroup (RS) that captures, by the primitive concept of (transversal) representation, an abstract order-theory of real spectra of a ring.

The present work generalizes the Witt ring concept of reduced special groups to real semigroups: for each RS, R, it is constructed its Witt ring, $W(R)$, and moreover, from $W(R)$, it is constructed the Von Neumann regular hull of R, $V(R)$. There exists a canonical "inclusion" morphism $i_R : R \to V(R)$ initial between the morphisms for R into Von Neumann RS: the map i induces properties similar to the Von Neumann regular hull of a ring and, in addition, there is canonical isomorphism between the Witt $W(R)$ and $W(V(R))$. These relations allows the analysis of the Witt ring of RS with tools available for RS Von Neumann as the description of the isometry of forms through a pp-formula and the characterization of the transversal representation, - in terms of isometry. As applications, we provide some calculation of the gradedWitt ring and an axiomatization of the Witt rings in a convenient language $L_{\omega_1,\omega}$.

▸ THANASES PHEIDAS, *Analogues of Hilbert's tenth problem for rings of power series, analytic and meromorphic functions.*
University of Crete, Department of Mathematics and Applied Mathematics, 70013 Heraklion, Greece.
E-mail: pheidas@uoc.gr.

We will give a survey on the question of decidability or undecidability of the existential problems (analogues of Hilbert;s tenth problem) for rings of formal power series, rings of analytic functions and fields of meromorphic functions. For example, consider the following question: Is there an algorithm which does the following: Given a polynomial equation $f(x) = 0$, with coefficients which are polynomials of the variable z over the integers and x is an array of variables, the algorithm determines whether the equation has solutions in any of the following domains:

a. Formal power series in z, with complex coefficients.

b. Formal power series in z, which have radius of convergence at least 1.

c. Same as in b. but with infinite radius of convergence.

Problem a. has a positive answer (S. Kochen et al). The similar problem in several variables has a negative answer (F. Delon). Problems b. and c. are open and seem out of the reach of current techniques. Similar problems exist in positive characteristic.

It is obvious that such questions, apart from being interesting on their own, may be important in applications, for example, in trying to determine the radii of convergence of complex valued functions that result from the study of natural phenomena.

We will survey survey work of, mainly but not only, R. Robinson, J. Ax, S. Kochen, F. Delon, A. Macintyre, V.d. Dries, J. Denef, L. Lipshitz, F.V. Kuhlmann, L. Rubel, H. Pasten, N. Garcia-Fritz and X. Vidaux.

▸ ANDRÉS VILLAVECES, *Some new infinitary logics and model theory.*
Departamento de Matemáticas, Universidad Nacional de Colombia, Bogotá, Colombia.
E-mail: avillavecesn@unal.edu.co.

The first logic (called L^1_κ for κ a singular strong limit cardinal) I will speak about was introduced by Saharon Shelah in 2012 (see [3]). The logic L^1_κ has many properties that make it very well adapted to model theory, despite being stronger than $L_{\kappa,\omega}$. However, it also lacks a good syntactic definition.
With Väänänen, we introduced the second logic (called $L^{1,c}_\kappa$, see [4]) as a variant of L^1_κ with a transparent syntax and many of the strong properties of Shelah's logic.
The third logic (called Chain Logic), while not new (it is due to Karp - see [2]), has been revisited recently by Dzamonja and Väänänen (see [1]) also in relation to Shelah's L^1_κ and the Interpolation property.

I will provide a description of these three logics, with emphasis on their relevance to model theory.

[1] Mirna Džamonja and Jouko Väänänen, *Chain Logic and Shelahâ's Infinitary Logic*, ***ArXiV 1908.01177***, August 2019.

[2] Carol R. Karp, *Infinite-quantifier languages and ω-chains of models*, ***Proceedings of the Tarski Symposium*** (University of California, Berkeley, Calif., June 23-30, 1971), (William Craig, C. C. Chang, Leon Henkin, John Addison, Dana Scott, and Robert Vaught, editors), vol. XXV, American Mathematical Society, 1979, pp. 225–232.

[3] Saharon Shelah, *Nice Infinitary Logics*, ***Journal of the American Mathematical Society***, vol. 25 (2012), no. 2, pp. 395–427.

[4] Jouko Väänänen and Andrés Villaveces, *A syntactic approach to Shelah's logic L^1_κ*, ***pre-print***, October 2019.

CONTRIBUTED TALKS

▸ JEAN-YVES BÉZIAU, *Abstract core of the completeness theorem.*
University of Brazil, Rio de Janeiro, Brasil.
E-mail: jyb@ufrj.br.
URL Address: www.jyb-logic.org.

In this talk we study the abstract core of the completeness theorem not depending on the specific language of a given logic which therefore is the kernel of many different systems of logic, following some previous works (e.g. [3]). This includes a generalized version of Lindenbaum-Asser lemma which is equivalent to the axiom of choice, as proved by W.Dzik [2]. We explain the relations at the abstract level between the completeness theorem, Lindenbaum-Asser lemma and the compactness theorem (a follow up of [1]). We then show how this general result can be applied to prove completeness theorems for a great variety of particular systems.

[1] J.-Y. Beziau, *La véritable portée du théorème de Lindenbaum-Asser,* ***Logique et Analyse****, vol. 167-168 (1999), pp. 341–359.*

[2] W. Dzik*, The existence of Lindenbaums extensions is equivalent to the axiom of choice,* ***Reports on Mathematical Logic****, vol. 13 (1981), pp. 29–31.*

[3] R. Goldblatt*, An abstract setting for Henkin Proofs,* ***Topoi****, vol. 3 (1984), pp. 37–41.*

▸ MARCO BARONE, NICOLÁS CARO, AND EUDES NAZIAZENO, *Uniform definability of integers in reduced indecomposable polynomial rings.*

Departamento de Matemática, Universidade Federal de Pernambuco, Avenida Jornalista Aníbal Fernandes, S/N - Recife/PE - Brasil.
E-mail: marco@dmat.ufpe.br.
Departamento de Matemática, Universidade Federal de Pernambuco, Avenida Jornalista Aníbal Fernandes, S/N - Recife/PE - Brasil.
E-mail: jorge.caro@dmat.ufpe.br.
Departamento de Matemática, Universidade Federal de Pernambuco, Avenida Jornalista Aníbal Fernandes, S/N - Recife/PE - Brasil.
E-mail: eudes@dmat.ufpe.br.

We prove first-order definability of the prime subring (that is, the image of the unique ring homomorphism from $\mathbb{Z}$) in reduced indecomposable (commutative, unital) polynomial rings, in the language of rings with signature $(0, 1, +, \cdot)$, by means of a unique formula. In characteristic zero, this defines integers in a class larger than that of polynomial integral domains, proving undecidability of the full theory of such rings and extending this way an old result by Raphael Robinson (1951). In order to achieve the result we build two distinct formulae working for two separate subclasses of our class of rings, and later manipulate and unify them into a single uniform formula. Our work explores issues such as: definability of sets of powers of fixed elements, a first-order exponent-extracting technique, quantification on suitable definable sets as a manner of reducing the language by removing the indeterminate, and the crucial relationship between polynomials and polynomial functions in our context.

▶ DIMITRA CHOMPITAKI, THANASES PHEIDAS, *Functional "Pell's equations" and analogues of Hilbert's tenth problem for certain rings of analytic functions.*
Mathematics and Applied Mathematics, University of Crete, Voutes Campus, Greece.
E-mail: d.hobitaki@gmail.com.
URL Address: math.uoc.gr/~hobitaki.
Mathematics and Applied Mathematics, University of Crete, Voutes Campus, Greece.
E-mail: pheidas@uoc.gr.

The first author was supported by the Hellenic Foundation for Research and Innovation (HFRI) and the General Secretariat for Research and Technology (GSRT), under the HFRI PhD Fellowship grant (GA. no. 4892).

At a glance: We examine the solutions of some functional "Pell's equations" in certain rings of analytic functions. In consequence, we prove that the positive existential theory of the ring of exponential sums is undecidable.

Define the set of *exponential sums*, EXP(C), where C is the symbol for the Complex Numbers, to be the the set of expressions

$$a = \alpha_0 + \alpha_1 e^{\mu_1 z} + \cdots + \alpha_N e^{\mu_N z}$$

where $\alpha_i, \mu_j \in C$. We ask whether the positive existential first order theory of EXP(C), as a structure of the language

$$\mathbf{L} = \{+, \cdot, 0, 1, \alpha\},$$

where α is a non constant function over EXP(C), is decidable or undecidable. In a recent unpublished paper P. D Aquino, Th. Pheidas and G. Terzo have had partial results in the direction of proving a negative answer (actually, a considerably more general statement) but they do it only pending on a number theoretic hypothesis. We provide a new proof, based partially on theirs, but using different tools ('Pell Equations' instead of Elliptic Curves). Our approach has been suggested by A. Macintyre. Our result may be considered as an analogue of Hilbert's Tenth Problem for this structure and as a step to answering the similar problem for the ring of exponential polynomials, which is still open.

[1] J. Denef , *Hilberts Tenth Problem for Quadratic Rings*, ***Transactions of the American Mathematical Society***, vol. 48 (1975), pp. 214–220.

[2] ——— *The diophantine problem for polynomial rings and fields of rational functions*, ***Transactions of the American Mathematical Society*** , vol. 242 (1978), pp. 391–399.

[3] L. van den Dries, *Exponential rings, exponential polynomials and exponential functions*, ***Pacific Journal of Mathematics***, vol. 113 (1984), pp. 51–66.

[4] Th. Pheidas and K. Zahidi, *Undecidable existential theories of polynomial rings and function fields*, ***Communications in Algebra***, vol. 27(10)(1999), pp. 4993–5010.

[5] P. DAquino, Th. Pheidas and G. Terzo, *Undecidability of the diophantine theory of exponential sums*, ***manuscript.***

▶ RAUL FIGUEROA, *Compact extensions of continuous logic and partition properties*.
Departamento de Matemáticas, Universidad de los Andes, Bogotá, Colombia.
E-mail: r.figueroa@uniandes.edu.co.

As shown in [2], continuous logic $\mathcal{CL}$ satisfies the analogue of Lindström's characterization of first order logic in terms of compactness and the downward Löwenheim-Skolem theorem. A natural question is whether there are proper extensions of $\mathcal{CL}$ closed under the continuous logical operations and satisfying some form of compactness. Many examples are known in the classical setting [1] but none in the continuous framework. For example, if Q_κ is the quantifier "there are κ many...", the logic $\mathcal{L}_{\omega\omega}(Q_{\omega_1})$ is a well known countably compact extensions of first order logic and $\mathcal{L}_{\omega\omega}(Q_{\mu^+})$ is compact for theories of power δ, if $\mu^\delta = \mu$. We introduce a notion of continuous generalized quantifier and show that for a continuous version of Q_κ the logic $\mathcal{CL}(Q_{\beth_{\omega_1}})$ is countably compact. More generally, $\mathcal{CL}(Q_{\beth_\delta})$ satisfies compactness for theories of power less than $cof(\delta)$. These and other related results depend on a combination of ultraproduct theorems and partition properties. This is joint work with Xavier Caicedo.

[1] J. Barwise and S. Feferman, editors. ***Model-theoretic logics***, Springer Verlag, 1985.

[2] X. Caicedo, *Maximality of continuous logic*, ***Beyond First-Order Model Theory*** (J. Iovino, editor), Chapman and Hall/CRC , 2017, pp. 106–128.

▸ DARÍO GARCÍA, *Pseudofinite structures, forking and unimodularity.*
Departamento de Matemáticas, Universidad de los Andes, Carrera 1 No. 18A-20, Colombia..
E-mail: dagarciar@gmail.com.

The fundamental theorem of ultraproducts (Łoś' Theorem) provides a transference principle between the finite structures and their limits. It states that a formula is true in the ultraproduct M of an infinite class of structures if and only if it is true for "almost every" structure in the class, which presents an interesting duality between finite structures and their infinite ultraproducts.

This kind of finite/infinite connection can sometimes be used to prove qualitative properties of large finite structures using the powerful known methods and results coming from infinite model theory, and in the other direction, quantitative properties in the finite structures often induce desirable model-theoretic properties in their ultraproducts. These ideas were used by Hrushovski (cf. [2] to apply ideas from geometric model theory to additive combinatorics, locally compact groups and linear approximate subgroups.

In this talk I will review the main concepts of pseudofinite structures, and present joint work with D. Macpherson and C. Steinhorn (cf. [1]) where we explored conditions on the (fine) pseudofinite dimension that guarantee good model-theoretic properties (simplicity or supersimplicity) of the underlying theory of an ultraproduct of finite structures, as well as a characterization of forking in terms of decrease of the pseudofinite dimension. I will also present the concept of unimodularity (for definable sets) - which is satisfied by both pseudofinite structures and omega-categorical structures - and joint with F. Wagner (cf. [3]) about the equivalence between difference notions of unimodularity.

[1] Darío García, Dugald Macpherson, Charles Steinhorn, *Pseudofinite structures and simplicity*, ***Journal of Mathematical Logic***, vol.15 (2015), no. 01, 1550002

[2] Ehud Hrushovski, *On pseudo-finite dimensions*, ***Notre Dame Journal of Formal Logic***, vol. 53 (2013), no. 3-4, pp. 463–495.

[3] Ehud Hrushovski and Frank Wagner, *Counting and dimensions*, ***London Mathematical Society Lecture Notes Series***, vol. 350 (2008), no. 161,

[4] Dugald Macpherson and Charles Steinhorn, *One-dimensional asymptotic classes of finite structures*, ***Transactions of the American Mathematical Society***, vol. 360 (2008), no. 1, pp. 411–448.

▸ SAMARIA MONTENEGRO, SILVAIN RIDEAU-KIKUCHI, *A generalisation of PAC, PRC and PpC fields.*

Departamento de matemática & Centro de investigación en matemática pura y aplicada, Universidad de Costa Rica, Sede Rodrigo Facio, San Pedro, San José, Costa Rica.
E-mail: samaria.montenegroguzman@ucr.ac.cr.
Mathematic Logic Team at Institut de Mathmatiques de Jussieu-Paris Rive Gauche, Paris Diderot University, Sophie Germain, 8 Place Aurlie Nemours, 75013 Paris, Francia.

The notion of pseudo algebraically closed field (PAC fields) has been generalized by Basarab and by Prestel to ordered fields, and by Grob, Jarden and Haran to p-adically valued fields. A field M is pseudo real closed (PRC) if M is existentially closed in every regular extension L to which all orderings of M extend. Analogously a field M is pseudo p-adically closed (PpC) if M is existentially closed in each regular field extension N to which all the p-adic valuations of M can be extended by p-adic valuations on N. We know that the complete theory of a PRC or a PpC field is NTP_2.

In this talk we will work with PRC and PpC fields and we are going to define a new class of fields that generalizes both of them, the pseudo T closed fields (PTC fields). The PTC fields contains the PRC and PpC fields, but also contains other fields as the PAC fields with one valuation.

The idea of the talk is to present some model theoretical properties of PTC fields, in particular we will show how we can extend the approximation property or the strong approximation property to these new class of fields.