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SET THEORY

Invited talks

Contributed talks

INVITED TALKS

▸ DANA BARTOSOVA, *Phase spaces of universal minimal flows.*
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We study actions of separable non-archimedean groups (equivalently groups of automorphisms of countable structures) on compact Hausdorff spaces. We call a continuous action of a topological group G on a compact Hausdorff space X a G-flow and we refer to X as the phase space of the action. We say that the action is minimal if the phase space has no non-trivial closed subset invariant under the action. The universal minimal flow of a group G is a minimal flow that has every minimal flow as a G-equivariant image. In the case of G non-archimedean, the phase space is always 0-dimensional and can therefore be studied in terms of its Boolean algebraof clopen sets. We study which Boolean algebras appear as algebras of clopen sets of phasespaces of (universal) minimal flows of non-archimedean groups.

▶ DIEGO MEJÍA, *Cichoń's maximum without large cardinals*.
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A very classical subject of study in combinatorics of the real line is Cichoń's diagram, which lists cardinal characteristics (and the provable relations between them) that encompasses combinatorial concepts related to measure and category of the real line, and to compactness of subsets of irrational numbers.

It is known that Cichoń's diagram is complete, in the sense that no other inequality between them can be proved. However, just in the present decade it has been explored consistency results regarding three or more different values considered simultaneously in the diagram.

Is it consistent with ZFC that all the cardinals in Cichoń's diagram are pairwise different? This is the main question of the research mentioned above.

A couple of years ago, Goldstern, Kellner and Shelah used large cardinals to answer this question affirmatively. Just very recently, the speaker joined their work, and they managed to show how to prove this consistency result without using large cardinals.

▸ ULISES ARIET RAMOS GARCÍA, *Open colorings on topological spaces.*
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In this talk, we shall present some results about the Todorčevićs Open Coloring Axiom ($OCA_{[T]}$) on topological spaces. This is joint work with José Antonio Corona-García.

▸ MARCIN SABOK, *Measurable Hall's theorem for actions of abelian groups.*
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I will discuss a measurable version of the Hall marriage theorem for actions of abelian groups. In particular, it implies that for free measure-preserving actions of such groups, if two equidistributed measurable sets are equidecomposable, then they are equidecomposable using measurable pieces. The latter generalizes a recent result of Grabowski, Máthé and Pikhurko on the measurable circle squaring and confirms a special case of a conjecture of Gardner. This is joint work with Tomasz Cieśla.

▸ TODOR TSANKOV, *Universal minimal flows of homeomorphism groups of high-dimensional manifolds.*
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The first interesting case of a non-trivial, metrizable universal minimal flow (UMF) of a Polish group was computed by Pestov who proved that the UMF of the homeomorphism group of the circle is the circle itself. This naturally led to the question whether a similar result is true for homeomorphism groups of other manifolds (or more general topological spaces). A few years later, Uspenskij proved that the action of a group on its UMF is never 3-transitive, thus giving a negative answer to the question for a vast collection of topological spaces. Still, the question of metrizability of their UMFs remained open and he asked specifically whether the UMF of the homeomorphism group of the Hilbert cube is metrizable. We give a negative answer to this question for the Hilbert cube and all closed manifolds of dimension at least 2, thus showing that metrizability of the UMF of a homeomorphism group is essentially a one-dimensional phenomenon. This is joint work with Yonatan Gutman and Andy Zucker.

CONTRIBUTED TALKS

▸ LUIS FELIPE BARTOLO ALEGRE, *A compatibility relation for sets.*
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In this lecture I generalize the logical concept of compatibility into a set theoretical one. The basic idea is that two sets are compatible if the *produce* at least one pair of opposite or contrary objects. The concept of compatibility depends, thus, on the concept of opposition. An operation of opposition $' : \mathbf{E} \longrightarrow \mathbf{E}$ needs to be *involutory* and, hence, bijective. Furthermore, an operation of opposition is anti-reflexive, in the sense that no x can be its own opposite. Some models of $'$ include the *inverse operation* of group theory, the *absolute complement* operation of set theory, some kinds of *antonymy*, and *classical negation*.

I define *classical compatibility* as a three-place relation $\mathsf{C} : \wp\mathbf{U} \times \wp\mathbf{U} \times \wp\mathbf{U}^{\wp\mathbf{U}}$ such that $\mathsf{C}(\mathbf{A}, \mathbf{B})^* \Leftrightarrow x, x' \in (\mathbf{A} \cup \mathbf{B})^*$, for no x. $\mathsf{C}(\mathbf{A}, \mathbf{B})^*$ is read "$\mathbf{A}$ is $*$-*compatible* with $\mathbf{B}$". C is *symmetric* in the sense that $\mathsf{C}(\mathbf{A}, \mathbf{B})^* \Leftrightarrow \mathsf{C}(\mathbf{B}, \mathbf{A})^*$ holds regardless of the $*$. However, C is no equivalence relation since it is neither *reflexive* nor *transitive*. Those properties clearly do not hold when $*$ is a closure operation.

[1] Batens, Diderik and Meheus, Joke, *The adaptive logic of compatibility*, ***Studia Logica***, vol. 66 (2000), no. 3, pp. 327–48.

▸ FRANQUI CÁRDENAS, *Inaccessible cardinals limit of weakly compact cardinals.*
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In [1] Menas prove that if κ is measurable cardinal limit of strongly compact cardinal then κ is strongly compact cardinal. We prove that if κ is an inaccessible cardinal such that the set of weakly compact cardinals below κ is an ineffable subset of κ then κ is weakly compact cardinal.

[1] MENAS TELIS, *On strong compactness and supercompactness*, ***Annals of mathematical logic***, vol. 7 (1975), no.4 pp. 327–359.

▸ CARLOS UZCÁTEGUI AYLWIN, *Combinatorial properties on extreme topological spaces with an analytic topology.*
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The study of maximal collections of subsets of a set X with some property has received some attention for a long time. Among the typical examples are the following families: maximal almost disjoint, maximal independent families, maximal filters and maximal topologies. When the base set X is countable, it is known that none of the above mentioned examples can be found analytic (as a subset of $\{0,1\}^X$ via characteristic functions). Maximal topologies are characterized as those which are *extremely disconnected* (i.e. the closure of every open set is open), *nodec* (i.e. every nowhere dense set is closed) and every open set is *irresolvable* (i.e. if U is open and $D \subseteq U$ is dense in U, then $U \setminus D$ not dense in U). There are neither extremely disconnected nor irresolvable spaces with an analytic topology. However, there are nodec analytic spaces.

A topological space X is *selectively separable* (SS), if for any sequence $(D_n)_n$ of dense subsets of X there is a finite set $F_n \subseteq D_n$, for $n \in \mathbb{N}$, such that $\bigcup_n F_n$ is dense in X. The existence of a maximal regular SS space is independent of ZFC. Thus it is natural to wonder about the status of SS on nodec spaces with an analytic topology. In this talk we review those results and present new examples.

This is a joint work with Javier Murgas.

▸ RAFAEL ZAMORA, *Borel colorings for directed graphs, a new approach.*
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In [1] they showed that an analytic graph either has a countable Borel coloring or it contains a copy of a particular graph G_0. This type of dichotomies have been explored by several authors since. For example, exploring a similar question for the class of analytic directed graphs that cannot be Borel-colored with 2 colors, B. Miller (unplublished) found that no such dichotomy can be found.

On the other hand, coloring graphs can be seen in a wider context as separation results of analytic sets on a plane. In this wider context, there are several new quasi-orders that have been explored by Lecomte (e.g. [2] & [3]), Zamora and others. We will give a new tool to transfer results from the graph context to the square one and vice-versa. Using this tool, we obtain the analogue of Miller's result for analytic subsets on a plane, as well as new proofs of several results.

[1] Kechris, A. S. and Solecki, S. and Todorcevic, S., *Borel chromatic numbers*, ***Advances in Mathematics***, vol.141 (1999), pp. 1–44.

[2] Lecomte, Dominique, *A dichotomy characterizing analytic digraphs of uncountable Borel chromatic number in any dimension*, ***Transactions of the American Mathematical Society***, vol. 361 (2009), no. 8 pp. 4181–4193.

[3] ——— *Potential Wadge classes*, ***Memoirs of the American Mathematical Society***, vol. 221 (2013), no.1038, pp.vi+83.

[4] Zamora, Rafael, *Separation of analytic sets by rectangles of low complexity*, ***Topology and its Applications***, vol. 182 (2015), pp.77–97.